UNIVERSITY of LIMERICK

OLLSCOIL LUIMNIGH

Faculty of Science and Engineering
Department of Mathematics & Statistics

# Special Mathematics Entrance Examination
# Ordinary Level

**DATE:** Thursday 21st August 2014

**TIME:** 14.30-17.30 (3 HOURS)

## INSTRUCTIONS TO CANDIDATES:

There are **two** sections in this examination paper.

| | | |
|---|---|---|
| Section A: | 6 questions, | 25 marks each. |
| Section B: | 3 questions, | 50 marks each. |

**ANSWER ALL QUESTIONS**

The invigilator will provide answer books, graph paper and a copy of the *Formulae and Tables* booklet. You must return it at the end of the examination. You are not allowed to bring your own copy into the examination.

You will lose marks if all necessary work is not clearly shown.

Answers should include appropriate units of measurement, where relevant.

Answers should be given in simplest form, where relevant.

Write the make and model of your calculator(s) here:

## Section A (6 Questions, 25 marks each)

1. (a) Solve the equations

$$4x + 3y = 5$$
$$3x - y = 7.$$

(b) Write as a power of 3

(i) $243$,

(ii) $\sqrt{27}$.

(c) Solve for $x$

$$3^{4x-1} = \left[\frac{243}{\sqrt{27}}\right]^2.$$

2. Two complex numbers are

$$u = 2 + 3i \quad \text{and} \quad v = 1 - i.$$

(i) Given that $w = u - 3v - 2$, write $w$ in the form $a + bi$.

(ii) Plot $u$, $v$ and $w$ on an Argand diagram.

(iii) Find a complex number $z$ in the form $a + bi$, such that

$$z = \frac{u}{v}.$$

3. The function $f$ is defined as

$$f : x \to x^3 - 3x^2 + 4, \quad x \in \mathbb{R}.$$

   (i) Find the coordinates of the point where the graph of $f$ cuts the $y$-axis.

   (ii) Find the coordinates of the local maximum and the local minimum turning points.

   (iii) Draw a rough sketch of the graph of $f$.

4. The points $A(-1, 2)$, $B(1, -2)$ and $C(7, 1)$ are the vertices of the triangle $ABC$.

   (i) Find the length of $AC$.

   (ii) Show that $AB$ is perpendicular to $BC$.

   (iii) Find the area of the triangle $ABC$.

   (iv) Find $|\angle BAC|$, correct to one decimal place.

5. (a) A circle has equation $x^2 + (y - 7)^2 = 100$.

(i) Write down the coordinates of the centre of the circle and its radius.

(ii) The point $(6, p)$ is on the circle. Find the two possible values of $p$.

(b) The circle $C$ has equation $x^2 + y^2 - 17 = 0$ and the line $l$ with equation $x - 4y - 17 = 0$ is a tangent to $C$ at the point $T$.

(i) Find the coordinates of $T$.

(ii) If the point $T$ is one end-point of a diameter of $C$, find the coordinates of the other end-point.

6. (a) (i) In how many different ways can a committee of four people be selected from ten people?

(ii) If one particular person must be on the committee, in how many ways can the committee be selected?

(b) Tickets for a raffle are placed in a box.
The box contains 15 blue tickets and 10 red tickets.
Tickets are drawn at random from the box and they are not replaced.
What is the probability that

(i) the first ticket drawn is red?

(ii) the first ticket drawn and the second ticket drawn are both red?

(iii) the first ticket drawn is red and the second ticket drawn is blue?

(iv) the first two tickets drawn are different in colour?

# Section B (3 Questions, 50 marks each)

7. A national convention centre hires its facility with the following cost structure:
Basic fee: €250,
Cost per delegate: €12.50,
The maximum capacity of the centre is 600.

   (i) Find an equation of the form $y = mx + c$ to represent the above cost structure where $y$ is the overall cost for $x$ delegates.

   (ii) Use the equation in (i) to find the cost of hiring the centre for 280 delegates.

   (iii) Graph the equation in (i) and show on the graph the result in part (ii).

   (iv) If a conference organiser received a bill of €4,250, use the graph in (iii) to estimate the number of delegates who attended the conference.

   (v) If, on negotiation, the management of the centre agreed a discount of $20\%$ on the individual cost per delegate for those delegates in excess of 250, calculate the savings for the conference organiser in (iv).

   (vi) How does the equation derived in (i) change to allow for the discount agreed in (v).

   (vii) Show graphically the relevant equations to describe the altered cost structure.

   [Note: There will now be two equations, one for $x \leqslant 250$, the other for $x > 250$.]

8. (a) John had a collection of old pennies. The following table shows how old each coin was and how much it weighted.

| **Age(years) x** | 51 | 47 | 53 | 33 | 39 | 46 | 42 | 48 | 28 | 36 |
|---|---|---|---|---|---|---|---|---|---|---|
| **Weight (grams) y** | 7.3 | 9.5 | 6 | 11.1 | 10.4 | 8.5 | 9.7 | 7.4 | 11.5 | 11.6 |

(i) Find the (average) mean age of the coins.

(ii) Find the (average) mean weight of the coins.

(iii) Draw a scatter graph to represent the data.

(iv) Comment on the type of correlation (if any).

(v) Plot the mean age and the mean weight point $(x, y)$ and label it $K$.

(vi) Draw a line of best fit through the mean age and mean weight point, $K$.

(b) Use a stem and leaf diagram (stemplot) to compare the examination marks in Physics and Chemistry for a class of 20 Leaving Certificate students.

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Physics | 75 | 69 | 58 | 58 | 46 | 44 | 32 | 50 | 57 | 77 |
| | 81 | 61 | 61 | 45 | 31 | 44 | 53 | 66 | 48 | 53 |
| Chemistry | 52 | 58 | 68 | 77 | 38 | 85 | 43 | 44 | 55 | 66 |
| | 65 | 79 | 44 | 71 | 84 | 72 | 63 | 69 | 79 | 72 |

Use the stemplots to find the median mark of Physics and the median mark of Chemistry.

9. (a) An open cylindrical can, without a lid, has a base radius of $r$ cm and a height of $h$ cm. The total outer surface area of the can is $300\pi$ cm$^2$.

(i) Express $h$ in terms of $r$.

(ii) Show that the volume $V$ cm$^3$ of the can is given by

$$V = 150\pi r - \frac{1}{2}\pi r^3.$$

(iii) Find the dimensions of the can that maximise the volume.

(iv) Calculate the maximum volume.

(b) A car begins to slow down at $P$ in order to stop at a red traffic light at $Q$. The distance of the car from $P$, after $t$ seconds, is given by

$$s = 12t - \frac{3}{2}t^2,$$

where $s$ is in metres.

(i) Find the speed of the car as it passes $P$.

(ii) Find the time taken to stop.

(iii) The car stops exactly at $Q$. Find the distance from $P$ to $Q$.