UNIVERSITY of LIMERICK

OLLSCOIL LUIMNIGH

Faculty of Science and Engineering

Department of Mathematics & Statistics

Special Mathematics Entrance Examination
Higher Level

DATE: Thursday 20 August 2015

TIME: 14.30-17.30 (3 HOURS)

INSTRUCTIONS TO CANDIDATES:

There are **two** sections in this examination paper.

Section A: 6 questions, 25 marks each.
Section B: 3 questions, 50 marks each.

ANSWER ALL QUESTIONS

The invigilator will provide answer books, graph paper and a copy of the *Formulae and Tables* booklet. You must return it at the end of the examination. You are not allowed to bring your own copy into the examination.

You will lose marks if all necessary work is not clearly shown.

Answers should include appropriate units of measurement, where relevant.

Answers should be given in simplest form, where relevant.

Write the make and model of your calculator(s) here:

SECTION A (6 questions, 25 marks each)

1. (i) Factorise the polynomial

$$f(x) = x^3 - 4x^2 - 11x + 30$$

(ii) Hence, solve the equation

$$x^3 - 4x^2 - 11x + 30 = 0$$

2. (i) Differentiate $2x^2 - \ln x$ with respect to x.

(ii) A curve has equation

$$y = 2x^2 - \ln x, \ x > 0.$$

Find the equation of the tangent to the curve at $x = 1$.

(iii) Find the coordinates of the turning point of the curve in (ii).

(iv) Determine the nature of the turning point in (iii).

3. The line $L : x + y = 12$ cuts the parabola $y = x^2$ at the point Q in the first quadrant.

   (i) Find the coordinates of Q

   (ii) Find the area enclosed by $x + y = 12$, $y = x^2$ and the x-axis.

4. Two fair six-sided dice are tossed.

   (i) Show the resulting sample space in a diagram.

   (ii) If x is the sum of the scores on the two dice, find

      (a) $P(x = 6)$,

      (b) $P(x > 6)$,

      (c) $P(x = 7|x > 5)$.

   (iii) Niamh plays a game where she tosses two dice.
   If the sum is 6, she wins €3.
   If the sum is greater than 6, she wins €1.
   If the sum is less than 6, she loses p euros.
   Find the value of p for which Niamh neither wins or loses.

5. A $6m$ high vertical wall casts a shadow of width $180cm$ on level ground. If Ann is $160cm$ tall, what is the maximum distance from the wall that she can stand and still remain entirely in the shade?

6. The circle with equation

$$x^2 + y^2 - 6x + 6y - 16 = 0$$

cuts the x-axis at the points A and B.

(i) Find the coordinates of the points A and B.

(ii) Find the equations of the tangents to the circle at the points A and B.

(iii) Show that the tangents found in (ii) above are not perpendicular to each other.

SECTION B (3 questions, 50 marks each)

7. A researcher wants to investigate whether the length of time people spend in education affects the income they earn. In a survey of twelve adults, they are asked to state their annual income and the numbers of years they spent in full-time education. The resultant data are given in the table below.

Years in Education	11	12	13	13	14	15	16	16	17	17	17	19
Income (€1,000)	28	30	35	43	55	38	45	38	55	60	30	58

(i) Identify the dependent and independent variables.

(ii) Represent the data in a scatter plot, showing the relevant axes.

(iii) Calculate the coefficient of linear correlation.

(iv) What can you conclude from your results in parts (ii) and (iii)?

(v) Add the line of best fit to the completed scatter plot in (ii).

(vi) Use the line of best fit to estimate the annual income of a person with 14 years full-time education.

(vii) By taking suitable readings from your diagram, or otherwise, calculate the slope of the line of best fit.

(viii) Explain how to interpret this slope in this context.

8. The approximate length of daylight in Limerick, measured in hours from sunrise to sunset in given by the function

$$f(t) = 12.29 + 4.75 \sin\left[\frac{2\pi}{365}t\right],$$

where t is the number of days after 21 March, and $\left[\frac{2\pi}{365}t\right]$ is expressed in **radians**.

(i) Find the length of daylight in Limerick on 1 June. Give your answer in hours and minutes, correct to the nearest minute.

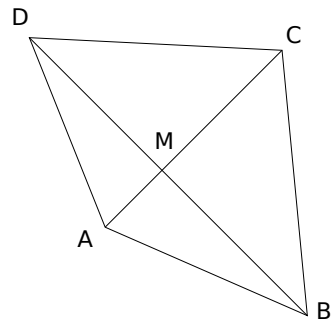
(ii) Find the date on which the length of daylight in Limerick is approximately 14 hours.

(iii) Find $f'(t)$, the derivative of $f(t)$.

(iv) Hence, or otherwise, find the length of the longest daylight time in Limerick.

(v) Using integration, find the average daylight hours in Limerick over the 92 day period from 21 March to 21 June. Give your answer in hours and minutes, correct to the nearest minute.

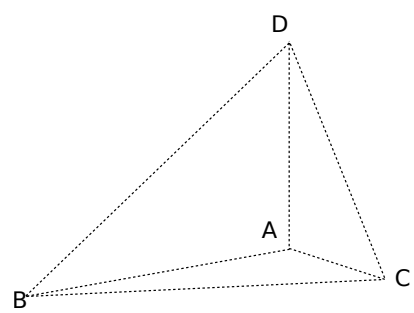
9. (a) $ABCD$ is a quadrilateral in which AC is perpendicular to BD.



(i) Why is $|AB|^2 = |AM|^2 + |BM|^2$?

(ii) Hence, prove that $|AB|^2 + |CD|^2 = |AD|^2 + |BC|^2$.

(b) A and B are two helicopter landing pads on level ground. C is another point on the same level ground.



$$|BC| = 800m, \ |AC| = 900m, \text{ and } |\angle BCA| = 60^o.$$

A helicopter at point D is hovering vertically above A.
A person at C observes the helicopter to have an angle of elevation of 30°.

(i) Find $|AD|$, in surd form.

(ii) Find $|BD|$.